Maths AA HL Questions:

Question 1:

[Maximum mark: 4]

Solve $\tan(2x-5°) = 1$

[for $0 < x < 180°$]

Answer Q1:

Since tan(x) is only 1 when the terminal angle is 45º, the angles used for (2x-5) are 45º and 225º. Hence, the answers are 25º **and** 115º.

Question 2:

[Maximum mark: 5]

$$\text{Solve } 3\cdot 9^x + 5\cdot 3^x - 2 = 0$$

Answer Q2:

Convert 9^x to 3^{2x} and then substitute 3^x into the equation. You will then get $3u^2 + 5u - 2 = 0$.

When solving the quadratic, you will get $u = 1/3$ and $u = -2$.

Then you substitute $u = 3^x$ back into the equation.

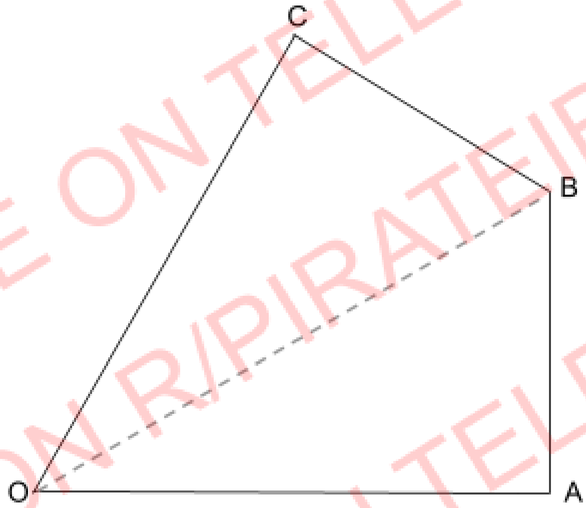
Since you cannot get -2 from an exponential function, you can eliminate -2.

Therefore, since $3^{-1} = 1/3$, the only answer is $x = -1$.

Question 3:

[Maximum mark:

The following diagram shows quadrilateral OABC.



Quadrilateral OABC is symmetrical across line OB.

Point C is $(3, 3\sqrt{3})$ and Point A is $(6, 0)$.

a) Find the midpoint of A and C.

b) The equation of the line OB.

Given that AB is perpendicular to OA,

c) Find the area of quadrilateral OABC.

Answer Key:

a) The average of 3 and 6 is 4.5 The average of 0 and $3\sqrt{3}$ is $(3\sqrt{3})/2$. The point is $(4.5, (3\sqrt{3})/2)$.

b) The gradient of line AC is $-\sqrt{3}$. AC and OB are perpendicular since OB is a line of symmetry. Hence, the gradient of line OB is $1/\sqrt{3}$. The intercept is 0, so no need to add anything.

c) OB is a line of symmetry, so find the area of triangle OAB and multiply by 2. Since the gradient of line OB is $1/\sqrt{3}$, the point B is $(6, 2\sqrt{3})$. Therefore, the distance AB is $2\sqrt{3}$. By using the formula $0.5 * (b * h)$, the area OABC is $12\sqrt{3}$.

Question 4:

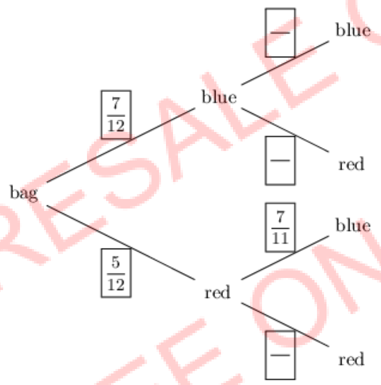
A species of bird has a different chance of nesting in different species.

In spring, the bird has a k chance of nesting.

In summer, the bird has a $k/2$ chance of nesting.

[Maximum mark: 6]
A bag contains 7 blue and 5 red marbles. Two marbles are selected at random without replacement.

1. Complete the tree diagram below. [3]



1. Find the probability that exactly one of the selected marbles is blue. [3]

a) Complete the tree diagram.

The probability of a bird not nesting in spring and not nesting in summer is 5/9.

b) Show that $9k^2 - 27k + 8 = 0$.

c) $9k^2 - 27k + 8$ is fulfilled by $k = 1/3$ and $k = 8/3$. Explain why $k = 8/3$ is not valid.

ANSWER:

a) The leftmost box should have $1-k$. Both boxes on the right should have $1 - k/2$.

b) $(1 - k)(1 - k/2) = 5/9$ then $(1 - 3k/2 + k^2/2) = 5/9$ then $(2 - 3k + k^2) = 10/9$ then $18 - 27k + 9k^2 = 10$ then $9k^2 - 27k + 8 = 0$

c) If $k = 8/3$ then the probability exceeds 1 which is not possible.

Question 5:

$f(x) = \frac{2(x+3)}{3(x+2)}$

a) What is the equation of the horizontal asymptote of the function?

$g(x) = mx + 1,\ m \neq 0$

b) If $m > 0$, how many solutions to $f(x) = g(x)$ are there?

c) For what value of m will there only be one solution?

d) What is the range of values for m for which $f(x) = g(x)$ has two solutions **when** $x \geq 0$?

ANSWER:

a) $f(x) = \frac{2x+6}{3x+6}$, so the horizontal asymptote is at $y = \frac{2}{3}$.

b) There will be two solutions.

c) -1/6

d) ?

ii) asks for what value of m will there be only one solution

I got it: I think its that the line mx + 1 is tangent to the curve so derivate. prove?

Question 6:

A farmer grows two different kinds of apples.

	Mean (g)	Standard Deviation
eating	100	20
cooking	140	40

For this question, you may assume that there are 95% of apples within two standard deviations.

a) What is the percentage of eating apples that weigh more than 140g?

This farmer grows 80% eating apples.

In addition, a sorter sorts all apples that weigh more than 140g into one container.

b) If you were to pick an apple out of this container, what is the probability that the apple you picked is an eating apple? Give your answer in $\frac{c}{d}$ where c and d are integers.

ANSWER:

a) 5% of apples are outside two STDEVs. Half of these apples are on the larger side. 2.5%

b) Cooking apples $> \frac{1}{2} \times \frac{1}{5} = \frac{1}{10} = \frac{10}{100}$, Eating apples $> \frac{1}{40} \times \frac{4}{5} = \frac{2}{100}$, so out of 100 apples, 12 of them will be greater than 140 grams and 2 of those 12 are eating apples. Therefore, in simplest terms, the probability is $\frac{1}{6}$ where c is 1 and d is 6.

Question 7:

Equation 1 is $2x^3 - 7x^2 + ...$

a) What is the sum of the 3 roots [ABC] of this equation?

Equation 2 is $2z^5 - 11z^4 + ... - 20$. Roots [ABC] in Equation 1 are also in Equation 2.

$h(z) = 0$ and $z = p + 3i$.

b) Show that p = 1.

It is given that h(0.5) = 0. A < B.

c) Find the value of AB.

d) Find the value of A and B separately.

ANSWER:

a) $\frac{-b}{a}$ gives you $\frac{7}{2}$.

b) $\frac{-b}{a}$ gives you $\frac{11}{2}$. Since there are 5 roots in Equation 2 and ABC are three of the roots, the last two roots have a sum of 2. In addition, since $z = p + 3i$ and all the coefficients are real, you can use CRT to say that $(p - 3i)$ and $(p + 3i)$ are the last two roots. Hence, you can say that $2p = 2$ and therefore $p = 1$.

c) The product of all five roots is $\frac{20}{2} = 10$. Since $h(0.5) = 0$, you can set root C as 0.5. Multiplying $(1 + 3i)$ and $(1 - 3i)$, you get 10. So, AB * 0.5 * 10 = 10. Therefore, AB = 2.

d) Since both A and B are integers and B is greater than A, A = 1 and B = 2.

Question 8:

Find $\lim_{h \to 0} \frac{sec^4(x) - cos^2(x)}{x^4 - x^2}$.

ANSWER:

-3?

Question 9:

A teacher, for safety, has to split *n* students into two groups.

The first group must have **exactly** 3 people. The second group must have **at least** 3 people.

a) Find the expression for the number of ways the groups can be sorted.

Two students can't work together and must be in different groups.

b) The number of ways the groups can be sorted is halved. Find the number of students *n*.

ANSWER:

?

-----Section B-----

Question 10:

[Maximum mark: 16]

An arithmetic sequence begins $a,\ p,\ q$.

a) Show that $2p - q = a$.

A geometric sequence begins $a,\ s,\ t$.

b) Show that $s^2 = at$.

It is given that $q = t = 1$ and $a \neq 0$.

c) Show that $p > \frac{1}{2}$.

It is given that $a = 9$, $s > 0$ and $q = t = 1$.

d) List the first four terms of the arithmetic sequence.

e) List the first four terms of the geometric sequence.

Another arithmetic sequence begins $9 + ln(9),\ 5 + ln(3),\ ...$

f) Find the common difference in this sequence.

g) Show that the sum of the first 10 terms is $-90 - 25\,ln(3)$.

ANSWER:

a) Let $p = a + d$ and $q = a + 2d$. Then substitute, $(2a + 2d) - (a + 2d) = a$.

b) Let $s = ar$ and $t = ar^2$. Then substitute, $a^2r^2 = a \times ar^2$.

c) $2p - 1 = a$, then $p = \frac{a+1}{2}$. Since a cannot be 0 or negative, $p > \frac{1}{2}$.

d) With simple arithmetic, $d = -4$. Therefore, it goes $9,\ 5,\ 1,\ -3$.

e) With simple arithmetic, $r = \frac{1}{3}$. Therefore, it goes $9,\ 3,\ 1,\ \frac{1}{3}$.

f) $(5 + ln(3)) - (9 + ln(9)) = (5 + ln(3)) - (9 + 2ln(3)) = -4 - ln(3)$.

g) Use the formula booklet. You get the right answer.

Question 11:

[Maximum mark: 19]

π_1: $2x + 6y - 2z = 5$ and point A is (2, ½, 1)

a) Show that point A lies in π_1.

π_2: $(k^2 - 6)x + (2k + 3)y + pz = q$, and $p = -6$.

b) Given that both planes are perpendicular and A lies in Plane 2, find *k* and *q*.

From part c onwards, Plane 1 and Plane 2 are parallel. It is also given that k=3.

c) Find *p*.

It is given that $q = \frac{51}{2}$.

d) A line perpendicular to Plane 1 that goes through A meets Plane 2 at a point B. Find B.

e) ?

ANSWER:

Answer currently incorrect, as point A was changed.

a) Substitute point A in the equation for Plane 1. It should come out equal.

b) Dot (2, 6, -2) with $((k^2 - 6), (2k + 3), -6)$ in order to get a long equation. This equation should equal zero. Simplifying gives $k^2 + 6k + 9 = 0$ which is just $(k + 3)^2$. Therefore, $k = -3$. Substitute the k value and the coordinates for A into the equation and get $q = \frac{3}{2}$.

c) Since Plane 1 and Plane 2 are parallel, the cartesian equation should be scalar multiples of each other. Therefore, since $k = 3$ and hence x = 3 and y = 9, p should be -3.

d) The resultant vector line is (2, ½, 1) + λ(2, 6, -2). Use the parametric form of this equation and substitute the variables into Plane 2's equation. Rearranging the equation gives

e)

f) Rearranging the equation gives $\lambda = \frac{25}{44}$. Use this value with the parametric equations to find point B $(\frac{138}{44}, \frac{172}{44}, \frac{-94}{44})$.

g) ?

Question 12:

maclaurin series with convergence and an induction

Original function f(x) = (1-ax)^-1/2

A: induction of the derivatives of a function

B: find the maclaurin of some function in A

C: show that it converges to..?

D: using x = 0.1, show the …………… of sqrt3

Hence, show

$$(-4x+1)^{-\frac{1}{2}}(-2x+1)^{-\frac{1}{2}} \approx \frac{19x^2+6x+2}{2}$$

Answer Q12:

(using proof by induction of the derivatives of a function)

SL:

Question 1: looking at the graph the questions were:

a)i) f(4)

ii)f(f(4))

iii) f-inverse(3)

b) sketch f-inverse(x)

Answer Q1:

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[for 0 < x < 180°]

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Question 3:

Answer Q3:

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Answer Q4:

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b) Convert 9^x to 3^{2x} and then substitute 3^x into the equation. You will then get $3u^2 + 5u - 2 = 0$.

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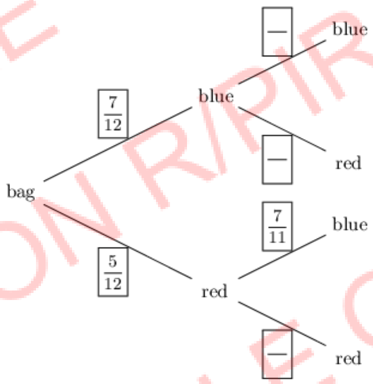

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Answer Q6: ANSWER:

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Question 7:

Answer Q7:

Question 8:

Answer Q8:

Question 9:

Answer Q9: